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Discrete mathematics

Boolean Algebra :- A Boolean Algebra of sets is a non-empty class $\mathcal{A}$ of subsets of the universal set U which has the following properties

(a) $A, B \in \mathcal{A} \Rightarrow A \cup B \in \mathcal{A}$

(b) $A, B \in \mathcal{A} \Rightarrow A \cap B \in \mathcal{A}$

(c) $A \in \mathcal{A} \Rightarrow A' \in \mathcal{A}$

$\mathcal{A}$ is assumed to be non-empty, so it ~~cot~~ contains at least one set A.

from property (c), A' is in $\mathcal{A}$ along with A.

$\because A \cap A' = \phi$ and $A \cup A' = U$

$\Rightarrow \mathcal{A}$ contains the empty set and the universal set.

Consider the collection C consisting only of the the empty set and the universal set.

obviously it is a Boolean algebra of sets.

These two distinct sets are the only ones which every Boolean algebra of sets must contain

Another Example

The class (or collection) of all subsets of U is also a Boolean algebra of sets.

Let $\mathcal{A}$ be a Boolean algebra of sets.

Let $\{A_1, A_2, \cdots, A_n\}$ is a non-empty finite subclass of $\mathcal{A}$. Then

$A_1 \cup A_2 \cup \cdots \cup A_n$ and $A_1 \cap A_2 \cap \cdots \cap A_n$ are both sets in $\mathcal{A}$.

Since $\mathcal{A}$ contains the empty set and the universal set.

$\Rightarrow \mathcal{A}$ is a class of sets which is closed under the formation of finite unions, finite intersections and complements.

Let $\mathcal{A}$ be a class of sets which is closed under the formation of finite unions, finite intersections and complements.

$\Rightarrow \mathcal{A}$ contains the empty set and the universal set.

i.e. $\mathcal{A}$ is non-empty

$\Rightarrow \mathcal{A}$ is a Boolean algebra of sets.

Hence, Boolean algebra of sets can be described alternatively as classes of sets which are closed under the formation of finite unions, finite intersections and complements.